

POMERON MODELS AND EXCHANGE DEGENERACY OF THE REGGE TRAJECTORIES

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Abstract

Two models for the Pomeron, supplemented by exchange-degenerate sub-leading Regge trajectories, are fitted to the forward scattering data for a number of reactions. By considering new Pomeron models, we extend the recent results of the COMPAS group, being consistent with our predecessors.

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1 Introduction

Ever since the discovery of the rising K^+p and π^+p total cross sections in Protvino [1], experimental data on hadronic total cross sections are among the most reliable tools for checking models of the Pomeron. The so-called "enhanced" cuts were among the first theoretical ideas used to explain the phenomenon [2, 3]. Since then, both the theory and experiment made enormous progress. Apart from hadronic reactions, measured now at much higher energies, photon-induced reactions also confirm the universal rise of the total cross sections. Empirical fits searching a universal Pomeron fitting all of the existing data were made e.g. in refs. [4, 5, 6].

The simplest models as the contribution to the total cross sections can be classified in three groups:

the dipole Pomeron: $\sigma \sim \log s$ [7, 8];

the "Froissart model": $\sigma \sim \log^2 s$ [9, 10];

the "supercritical" Pomeron: $\sigma \sim s^\epsilon$ [11, 12, 13] (actually, the list of relevant references is much longer and we apologize to those omitted there).

The last option, the supercritical one, in QCD corresponds to an infinite ladder of reggeized gluon exchange [14]. At finite energies, however only a finite number of diagrams contributes, giving rise to a finite series in logarithms of s [15] like

$$\sigma \sim \sigma_0 + \sigma_1 \log s + \sigma_2 \log^2 s, \quad (1)$$

respecting the Froissart bound. In the language of Regge-poles such a finite series corresponds to multipoles [16].

On the other hand, an obvious way to generalize [5] the Donnachie-Landshoff model [13] is to add another constant, whenafter it becomes

$$\sigma \sim \sigma_0 + \sigma_1 s^\epsilon. \quad (2)$$

In ref. [17] the model (2) was interpreted as that corresponding to two Pomeron poles.

Recently a detail analysis of three Pomeron models, namely

$$\text{I. } P \sim \sigma_0 s^\epsilon, \quad (3)$$

$$\text{II. } P \sim \sigma_0 + \sigma_1 \log s, \quad (4)$$

$$\text{III. } P \sim \sigma_0 + \sigma_2 \log^2 s, \quad (5)$$

was performed. Besides the Pomeron non-exchange-degenerate sub-leading contributions were included. The intercepts of the f/a and ω/ρ Regge trajectories were assumed to be different.

The analysis [6] is the most complete and professional one among those existing. It shows e.g. that the lowest energy, where the above parametrization works (i.e. $\chi^2/dof \sim 1$) is $\sqrt{s} = 9 \text{ GeV}$.

In the present paper we continue the program initiated by [6] by scrutinizing two more earlier unexplored models for the Pomeron. We show that they do not require the secondary trajectories to be exchange-degenerate. This disputable point was recently raised also in ref. [17]. As seen from Fig. 1 the 10 resonances belonging to the 4 different $I^G(J^{PC})$ families $\rho - \omega - f_2 - a_2$ are compatible with a unique linear exchange-degenerate Regge trajectory with $\alpha(0) = 0.48$ intercept.

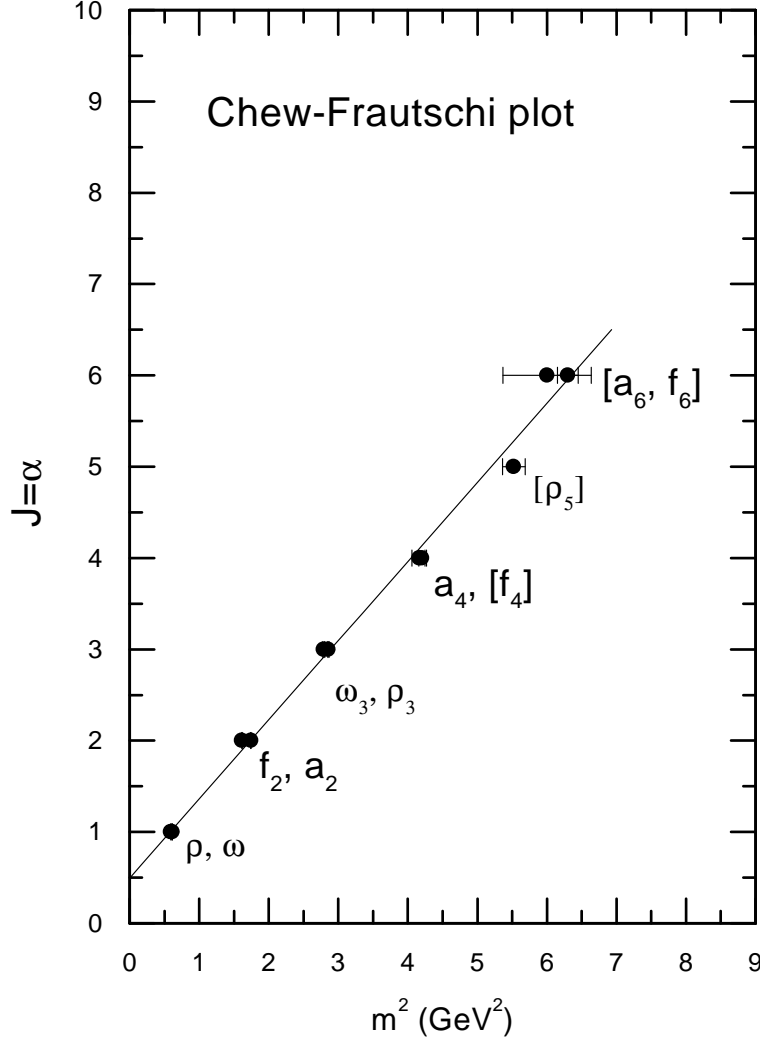


Fig. 1. The ρ - ω - f_2 - a_2 exchange-degenerate Regge trajectory from the resonance masses (Particle Data Group).

Our procedure follows the paper [6] as close as possible. Firstly, we use the same data sample [20], secondly, we use the same number of free parameters (namely, 16), thirdly, we calculate the errors in the same way as the authors of ref. [6] do. Moreover, we use the same notation. This universality makes it possible to confront the resulting fits without any bias.

2 Alternating series for the Pomeron

Let us write the real and imaginary parts of the forward elastic scattering amplitude as

$$\frac{1}{s} \text{Im} A_{h_1 h_2}(s) = \text{Im} P(s) + \text{Im} R(s), \quad (6)$$

$$\frac{1}{s} \text{Re} A_{h_1 h_2}(s) = \text{Re} P(s) + \text{Re} R(s), \quad (7)$$

where

$$ImP(s) = \lambda_{h_1 h_2} \left[A + B \log \left(\frac{s}{s_0} \right) + C \log^2 \left(\frac{s}{s_0} \right) \right], \quad (8)$$

$$ReP(s) = \frac{\pi}{2} \lambda_{h_1 h_2} B + \pi \lambda_{h_1 h_2} C \log \left(\frac{s}{s_0} \right), \quad (9)$$

comes from the Pomeron contribution and

$$ImR(s) = \left[Y_1^{h_1 h_2} \mp Y_2^{h_1 h_2} \right] s^{-\eta}, \quad (10)$$

$$ReR(s) = - \left[Y_1^{h_1 h_2} \cot \left(\frac{1-\eta}{2} \pi \right) \pm Y_2^{h_1 h_2} \tan \left(\frac{1-\eta}{2} \pi \right) \right] s^{-\eta}, \quad (11)$$

corresponds to the contribution of two exchange-degenerate Regge trajectories as in the classical paper of Donnachie and Landshoff [4]. Everybody is aware of the fact that exchange degeneracy is violated. The question is only the amount of this violation and the price to be paid for the introduction of additional free parameters. The quality of our fits show (see below) that exchange degeneracy is a reasonable approximation to reality.

We have fitted the above model to the data on total cross sections and the ratio of the real to imaginary part of pp , $\bar{p}p$, $\pi^\pm p$, $K^\pm p$, γp and $\gamma\gamma$ scattering, starting from $\sqrt{s_{\min}}$ until the highest available energies. As in ref. [6], we have studied the stability of our fit by varying the lower limit of the fit $\sqrt{s_{\min}}$ between 3 and 13 GeV . The number of the experimental data points and the resulting fit (value of χ^2/dof) as well as the dependence of the fitted parameters on the lower bound $\sqrt{s_{\min}}$ are presented in Fig. 2.

The results of our fits basically are consistent with those of ref. [6]. The minor difference is in the limiting value $\chi^2/dof \sim 1$, reached at $\sqrt{s_{\min}} = 10$ GeV in our fits (instead of 9 of ref. [6]). The difference obviously comes from exchange degeneracy of the subleading trajectories imposed here but relaxed in [6]. The fits for the total cross sections and ρ -ratio (for $\sqrt{s_{\min}} = 10$ GeV) extrapolated to $\sqrt{s_{\min}} \geq 3$ GeV , are shown in Fig. 3.

An important result of our fit is that the coefficients of the series (1) have alternating signs (see Table 1). This property and the rapid decrease of their absolute values (as $\sim 1/10$) provides the fast convergence of the series and ensures the applicability of this approximation at still much higher energies. The physical motivation of such a finite series representation of the Pomeron was discussed earlier - both in the context of Regge multipoles [19, 16] and QCD [15].

| A (mb) | B (mb) | C (mb) | η | $\chi^2/d.o.f.$ | statistics |
|----------------|------------------|---------------------|---------------------|---------------------------|-------------------------------|
| 37.1 ± 1.6 | -1.55 ± 0.33 | 0.275 ± 0.018 | 0.497 ± 0.019 | 1.03 | 314 |
| | pp | πp | Kp | $\gamma p \times 10^{-3}$ | $\gamma\gamma \times 10^{-4}$ |
| λ | 1 | 0.6248 ± 0.0019 | 0.5502 ± 0.0029 | 3.068 ± 0.050 | 0.0822 ± 0.0059 |
| Y_1 (mb) | 49.5 ± 4.7 | 12.0 ± 2.4 | -6.2 ± 2.8 | 44 ± 26 | 0.5 ± 4.2 |
| Y_2 (mb) | 25.1 ± 2.9 | 5.72 ± 0.69 | 10.5 ± 1.2 | | |

Table 1: Values of the fitted parameters in the alternating series Pomeron model (6)-(11), with a cut-off $\sqrt{s} = 10$ GeV and $s_0 = 1$ GeV^2 fixed.

3 Two-component Pomeron

The main virtue of the D-L model of the Pomeron [4] is its simplicity. Although it the single-term (factorizable!) D-L model (appended by a nonleading term) fits the data remarkably well [4], its possible extensions involving more free parameters are obvious and were discussed by various authors [6, 18]. One possibility is to add a constant term, resulting in the following expression for the Pomeron contribution [5, 17]

$$P \sim \sigma_0 + \sigma_1 s^\varepsilon. \quad (12)$$

Written as

$$ImP(s) = \lambda_{h_1 h_2} [A + Bs^\delta], \quad (13)$$

$$ReP(s) = -\lambda_{h_1 h_2} Bs^\delta \cot\left(\frac{1+\delta}{2}\pi\right), \quad (14)$$

the model has the same number of free parameters as in the previous case (Section 2) or in ref. [6]. Similar to the previous case, see eqs. (10) and (11), the model is appended by a pair of exchange-degenerate sub-leading trajectories.

The optimal value of $\chi^2/dof \sim 1$, as in the previous case, is reached when $\sqrt{s_{\min}} = 10 \text{ GeV}$.

The resulting fits are shown in Fig. 4. The fits for the total cross sections and ρ -ratio for $\sqrt{s_{\min}} = 10 \text{ GeV}$, extrapolated in the same way as for the previous case (see Section 2), are shown in Fig. 5.

Table 2 quotes the values of the fitted parameters with their errors calculated with the "threshold value" $\sqrt{s_{\min}} = 10 \text{ GeV}$. A pair of exchange-degenerate sub-leading trajectories was also included.

| ϵ | $A \text{ (mb)}$ | $B \text{ (mb)}$ | η | $\chi^2/\text{d.o.f.}$ | statistics |
|---------------------|------------------|---------------------|---------------------|---------------------------|-------------------------------|
| 0.1286 ± 0.0092 | 18.7 ± 2.3 | 8.3 ± 1.4 | 0.480 ± 0.018 | 1.09 | 314 |
| | pp | πp | Kp | $\gamma p \times 10^{-3}$ | $\gamma\gamma \times 10^{-4}$ |
| λ | 1 | 0.6253 ± 0.0019 | 0.5514 ± 0.0030 | 3.068 ± 0.051 | 0.0822 ± 0.0061 |
| $Y_1 \text{ (mb)}$ | 64.2 ± 4.0 | 22.8 ± 1.5 | 4.5 ± 1.8 | 100 ± 22 | 2.1 ± 3.8 |
| $Y_2 \text{ (mb)}$ | 22.5 ± 2.4 | 5.3 ± 0.6 | 9.5 ± 1.0 | | |

Table 2: Values of the fitted parameters in the two-component Pomeron model (6), (7), (10), (11), (13) and (14), with a cut-off $\sqrt{s} = 10 \text{ GeV}$ and $s_0 = 1 \text{ GeV}^2$ fixed.

4 Conclusions

We have fitted two hitherto unexplored models for the Pomeron to the data and compared the results with other fits [6]. We find that:

1. Exchange degeneracy is a good approximation to reality;
2. The series (1) has alternating signs;
3. The quality of the models fitted is not inferior to similar ones with the same number of free parameters.

4. Both ε and η correspond to those explored in [17].

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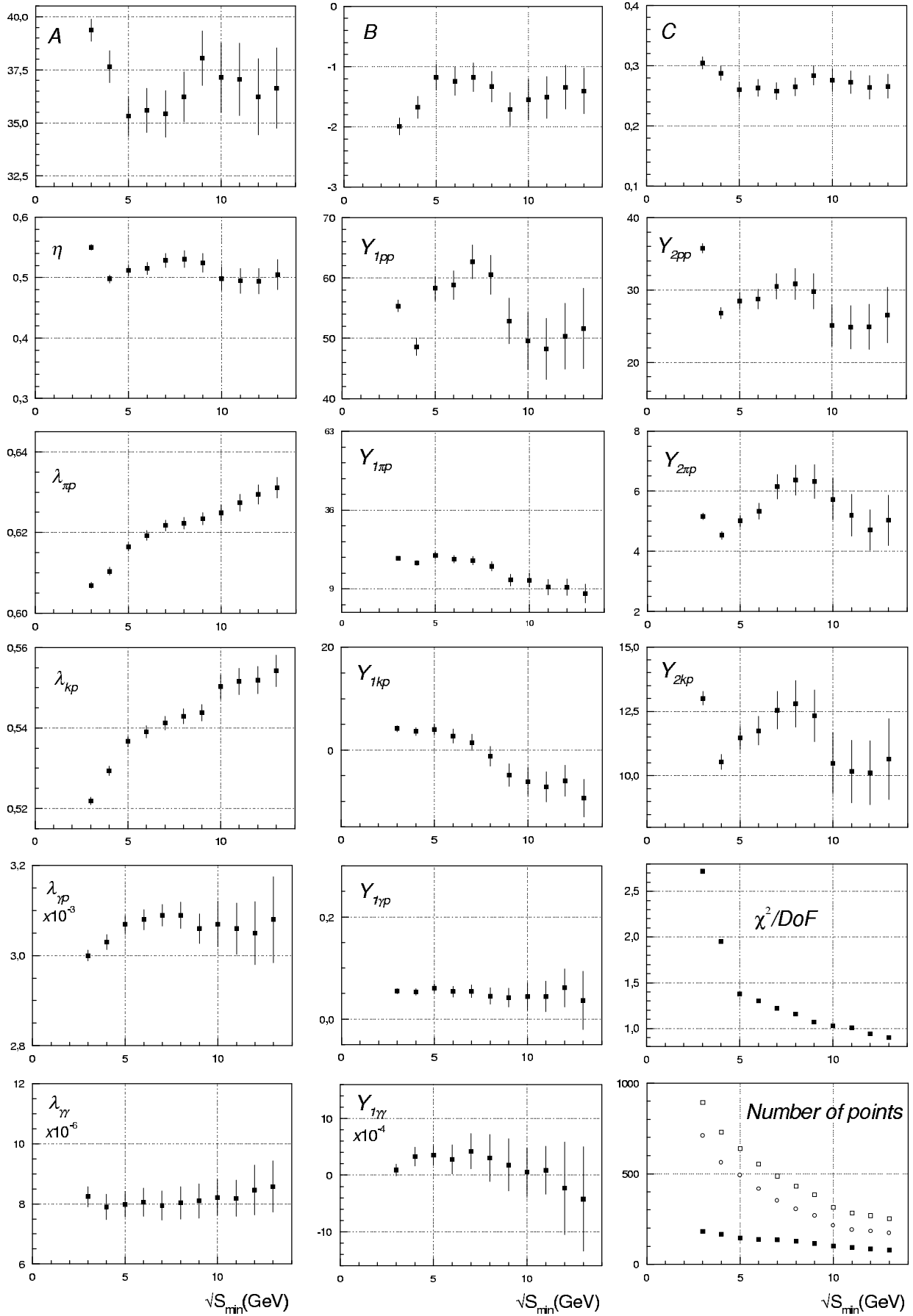


Fig. 2. Parameters of the alternating series Pomeron model as functions of the minimum energy considered in the fit.

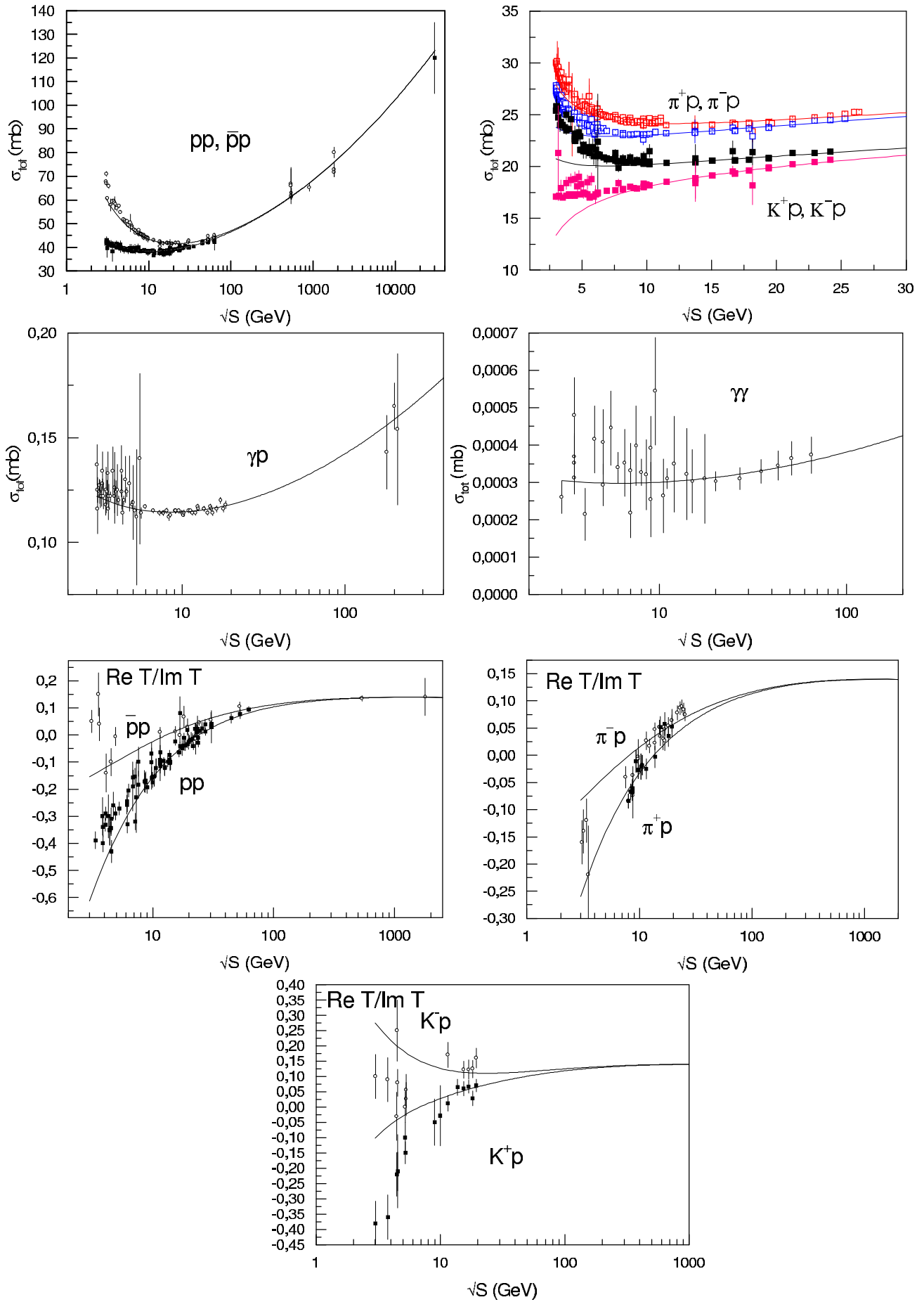


Fig. 3. Fit to the total cross sections and to the p -ratio from parametrization (6)-(11).

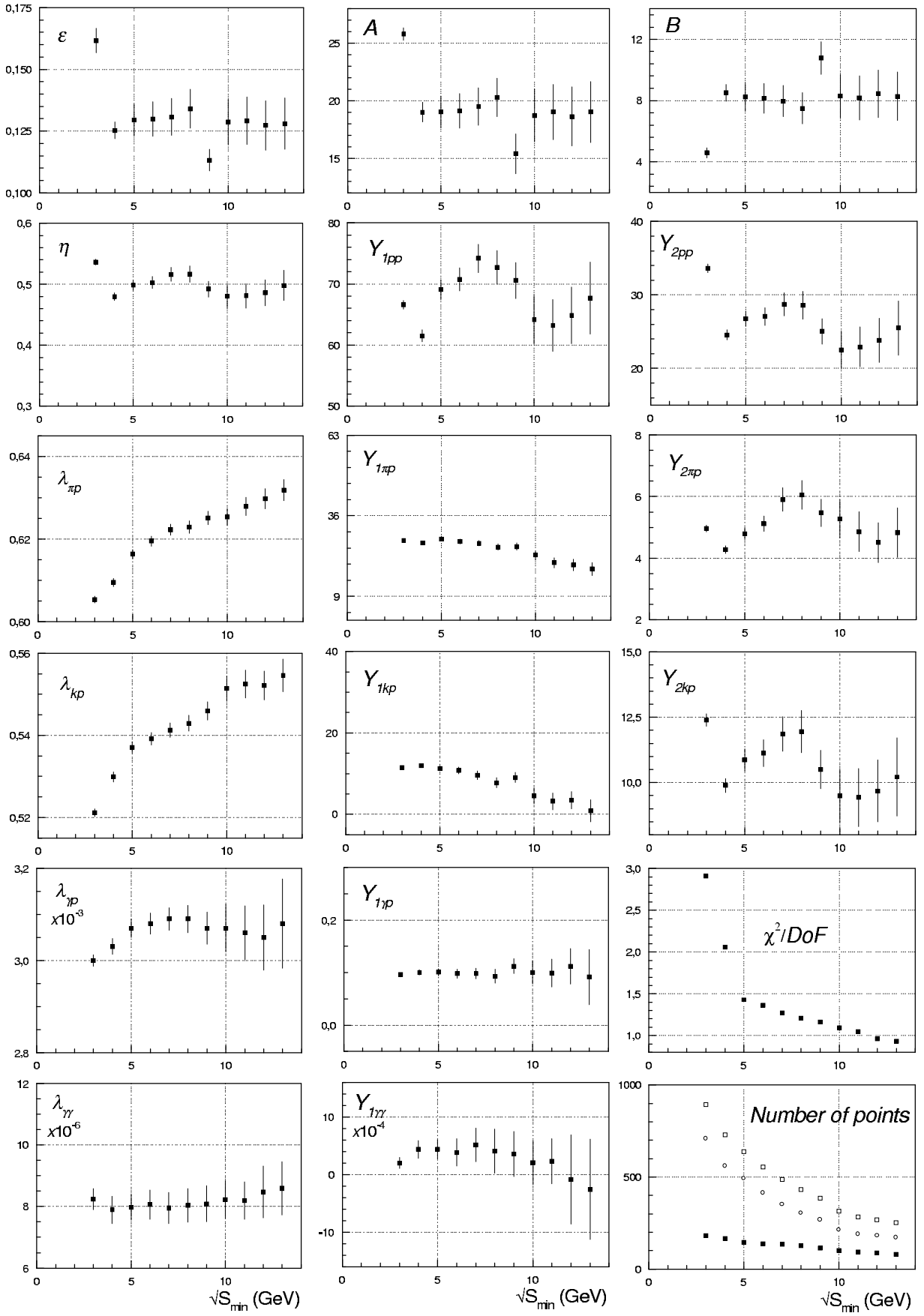


Fig. 4. Parameters of the two-component Pomeron model as functions of the minimum energy considered in the fit.

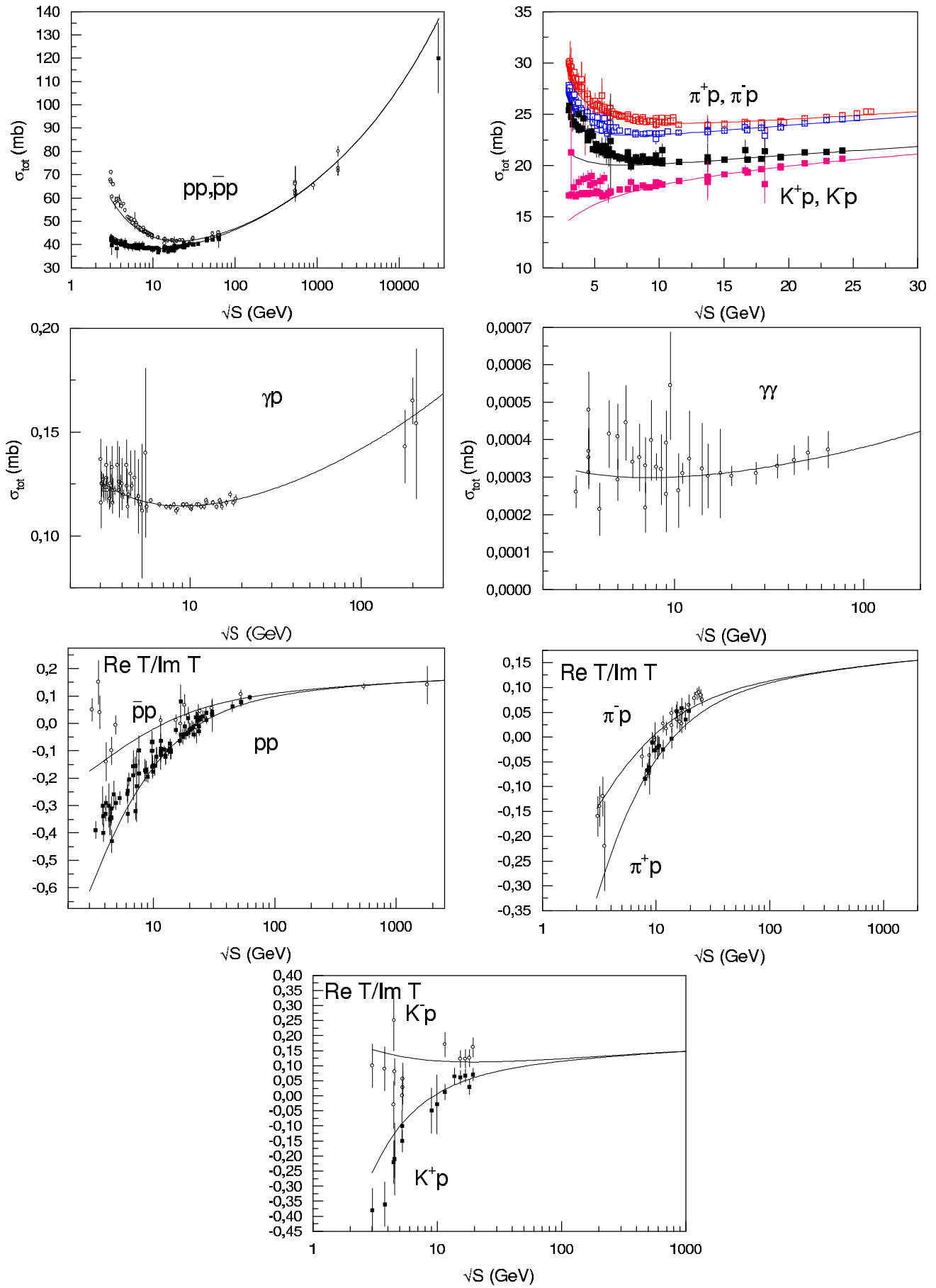


Fig. 5. Fit to the total cross sections and to the ρ -ratio using the parametrizations (6), (7), (10), (11), (13) and (14).